Towards optimal dynamical models of the Milky Way halo mass

–Why satellites are better dynamical tracers

Jiaxin Han

Department of Astronomy, Shanghai Jiao Tong University

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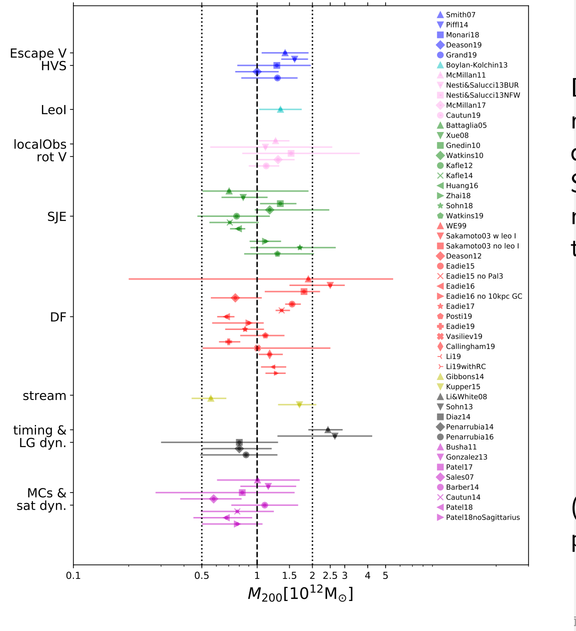
References

arxiv:1502.03477, 1507.00769, 1507.00771, 1605.09386, 1801.07373, 1909.02690, 1910.11257,..

Collaborators

Wenting Wang (IPMU), Zhaozhou Li (SJTU)
Shaun Cole (Durham), Carlos Frenk, et al.

Milky Way mass divergence



Different measurements disagree by $\sim \times 4$: Systematics! in the modelling of the tracers

(Wang,Han+19, in prep)

Steady-state methods

- time independent tracer distribution function (DF)

$$P_{\psi}(\vec{x}, \vec{v}) \Rightarrow \psi$$

  - Jeans theorem:

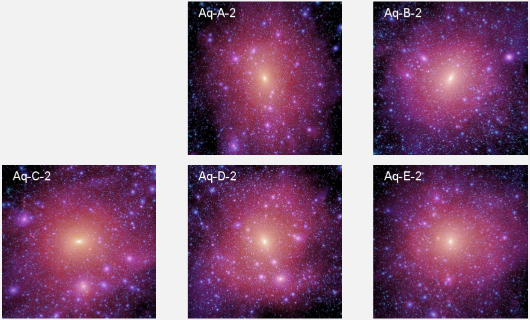
$$\frac{\partial P}{\partial t} = 0 \Leftrightarrow P(\vec{x}, \vec{v}) = f(J_1, J_2, J_3 ...)$$

    - $J_1, J_2, J_3 ...$: integrals of motion
    - additional assumptions about functional form required

Testing a state-of-the-art $f(E, L)$ method

$$
\left.\begin{aligned}
& f(E,L) = L^{-2\beta} F(E) \\
& \text{NFW potential } (M, c) \\
& \int f(E,L) d^3v = \rho(r)
\end{aligned}\right\} \Rightarrow
$$

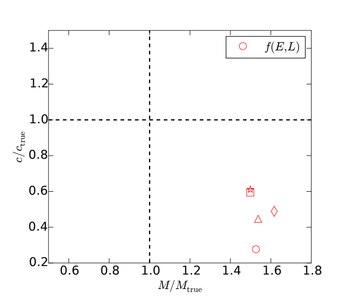
$$
\begin{aligned}
& P(r, v_r, v_t | \rho_s, r_s, \beta, \alpha, \gamma, r_0) = f(E,L) = \\
& -\frac{r_s^{-\alpha-\gamma} l^{-2\beta}}{2^{3/2-\beta}\pi^{3/2} v_s^3 \Gamma(\beta+1/2)\Gamma(1-\beta)} \times \int_{R_{\rm inner}}^{R_{\rm max,t}} dR(\epsilon - \phi(R))^{\beta-1/2} \times \\
& \left\{ \frac{(2\beta+1)R^{2\beta}\left(\frac{R}{1+R} - \ln(1+R)\right) - \left[\frac{1}{(1+R)^2} - \frac{1}{1+R}\right] R^{2\beta+1}}{\left[\frac{R}{1+R} - \ln(1+R)\right]^2} \times \frac{(2\beta-\alpha)\left(\frac{R}{r_0}\right)^{\alpha} r_s^{-\gamma} + (2\beta-\gamma)\left(\frac{R}{r_0}\right)^{\gamma} r_s^{-\alpha}}{\left[\left(\frac{R}{r_0}\right)^{\alpha} r_s^{-\gamma} + \left(\frac{R}{r_0}\right)^{\gamma} r_s^{-\alpha}\right]^2} + \right. \\
& \frac{R^{2\beta+1}}{\left[\frac{R}{1+R} - \ln(1+R)\right]\left[\left(\frac{R}{r_0}\right)^{\alpha} r_s^{-\gamma} + \left(\frac{R}{r_0}\right)^{\gamma} r_s^{-\alpha}\right]^2} \times \left[(2\beta-\alpha) r_s^{-\alpha-\gamma}\left(\frac{\alpha}{r_0} - \frac{2\gamma}{r_0}\right)\left(\frac{R}{r_0}\right)^{\alpha+\gamma-1} + \right. \\
& \left.\left. (2\beta-\gamma) r_s^{-\alpha-\gamma}\left(\frac{\gamma}{r_0} - \frac{2\alpha}{r_0}\right)\left(\frac{R}{r_0}\right)^{\alpha+\gamma-1} - (2\beta-\alpha) r_s^{-2\gamma}\frac{\alpha}{r_0}\left(\frac{R}{r_0}\right)^{2\alpha-1} - (2\beta-\gamma) r_s^{-2\alpha}\frac{\gamma}{r_0}\left(\frac{R}{r_0}\right)^{2\gamma-1}\right]\right\}
\end{aligned}
$$

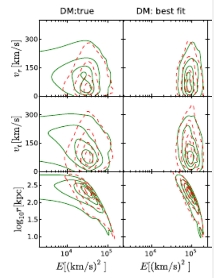


Fit $P_\psi(x, v)$ to Aquarius halos

Testing a state-of-the-art $f(E, L)$ method

$$\left.\begin{array}{l} f(E,L) = L^{-2\beta} F(E) \\ \text{NFW potential } (M, c) \\ \int f(E,L) d^3 v = \rho(r) \end{array}\right\} \Rightarrow P(x, v|\psi(M, c))$$





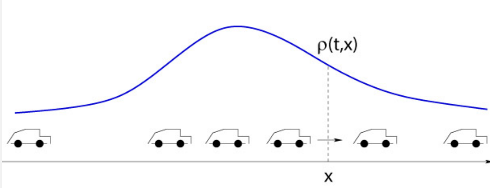
The fits are biased!

- fail to describe the loosely-bound particles

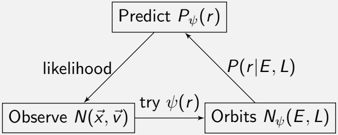
the orbital Probability Distribution Function (oPDF)

Steady-state solution to collisionless Boltzmann equation:

$$dP(x|\mathrm{orbit}) \propto dt$$



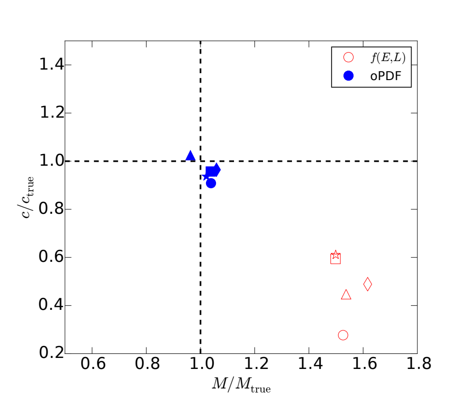
$$dP(r|E,L) = \frac{dr}{v_r(E,L,r)T(E,L)}$$



(Han+16a)

oPDF: Fits to Aquarius haloes

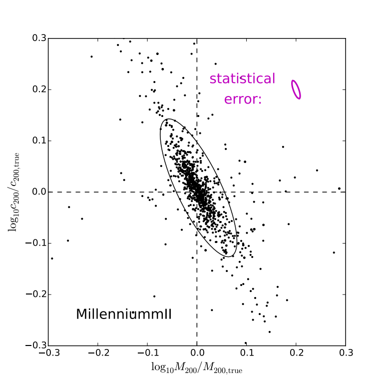
Tests on haloes from cosmological simulations:



(Han+16b)

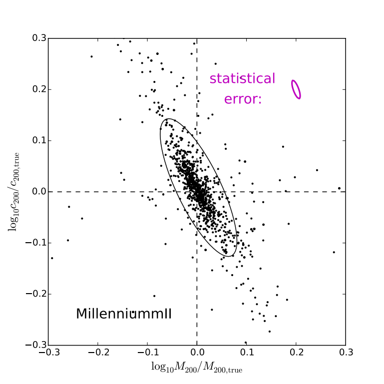
- no global systematic bias using oPDF: main source of bias removed
- still significant and *correlated* individual biases?

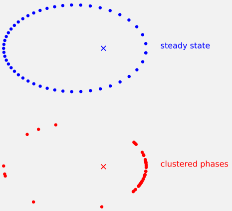
oPDF: Fits to many haloes



- Ensemble-unbiased
- Significant irreducible individual bias
  - $\sigma_M \sim 0.1$ dex (20%) for DM
  - correlate similarly as the statistical noises

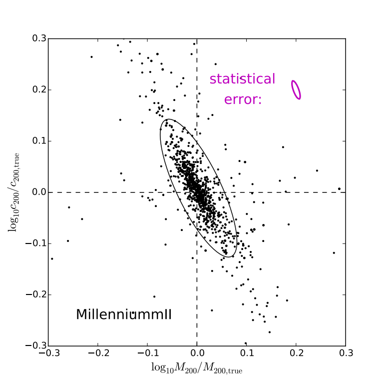
oPDF: Fits to many haloes

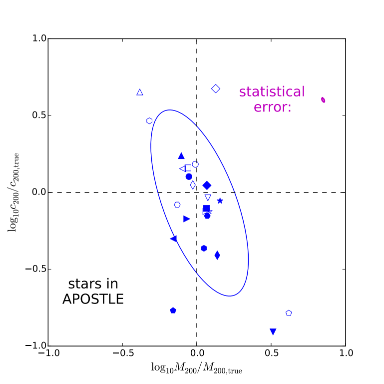




- Ensemble-unbiased
- Significant irreducible individual bias
  - $\sigma_M \sim 0.1$ dex (20%) for DM
  - correlate similarly as the statistical noises
  - Interpretation: correlated phase-space structure reduces N_{eff}

oPDF: Fits to many haloes

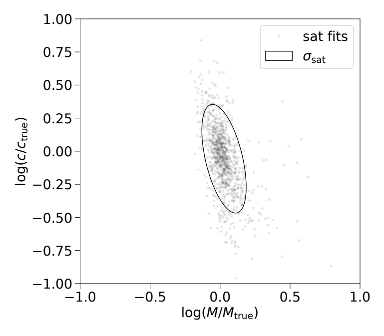




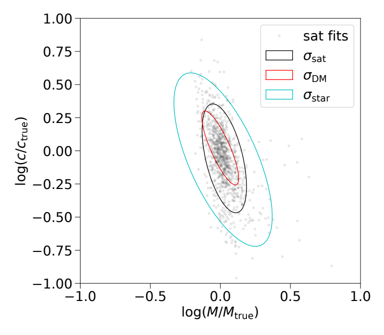
(Wang,Han+17)

- Stars are less in equilibrium than DM
  - DM: $\sigma_M \sim 0.1$ dex (20%)
  - Star: $\sigma_M \sim 0.3$ dex ($\sim \times 2$)
- Ubiquitous: Almost identical results from Jeans modelling (Wang, Han+18)

Fits to Satellite Galaxies



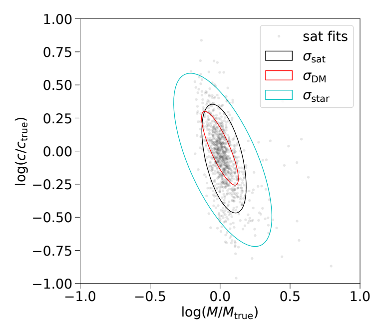
Fits to Satellite Galaxies

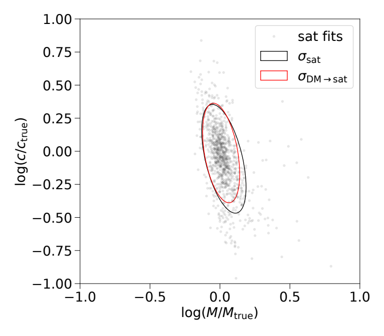


- Satellites are better tracers than stars
- Dynamical state of satellite tracers are close to DM

Han+2019

Fits to Satellite Galaxies





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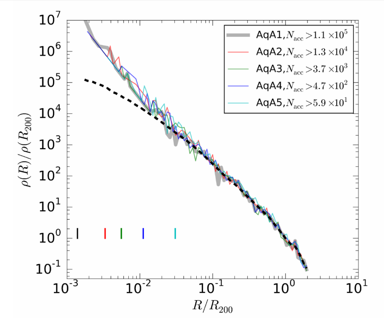
Han+2019

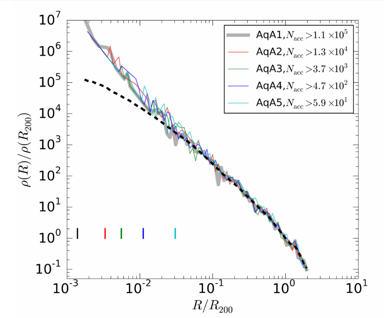
Satellites as unbiased dynamical tracers



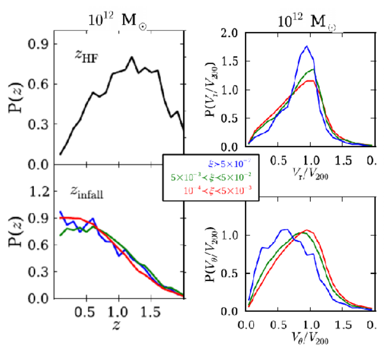
(Han+,2016c)

- Satellites follow the spatial distribution of DM
- Satellites follow the orbital distribution of DM

Satellites as unbiased dynamical tracers



(Han+,2016c)

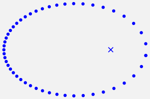


(Jiang+2015)

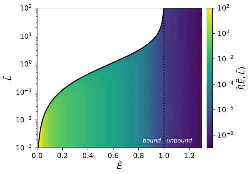
- Satellites follow the spatial distribution of DM
- Satellites follow the orbital distribution of DM

Going beyond the steady-state information

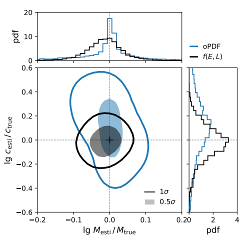
steady-state: $P_\psi(r|E,L)$



orbits: $P_\psi(E,L)$



full DF: $P_\psi(r,E,L) = P(r|E,L)P(E,L)$



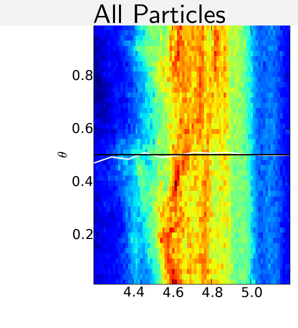
Li, Qian, Han+2019

See Zhaozhou Li's talk!

Summary and Conclusions

- Dynamical models have to be carefully designed for the system
  - easily biased by unjustified assumptions
  - pure steady-state methods can avoid these unncessary biases
- Simulated haloes are approximately steady-state systems
  - correlated phase-space structure violates steady-state assumption, leading to irreducible stochastic bias
    - Dynamical information exhausted with ~ 1000 DM particles or ~ 50 stars
    - Intrinsic $\sigma_M \sim 20\%$ (DM) or $\sim \times 2$ (stars): the information limit of steady-state modelling
- Satellite galaxies are better tracers than stars, with a dynamical state close to DM particles: $\sigma_M \sim 25\%$ with around 1000 satellites.
  - Going beyond the steady-state information can further improve dynamical inference

What determines the systematics?



- The phase distribution is only approximately uniform (steady-state).

$$\theta(r, \mathrm{orbit}) = \frac{t}{T}, \quad dP(\theta|\mathrm{orbit}) = d\theta$$

- There are correlated structures associated with substructures and streams.

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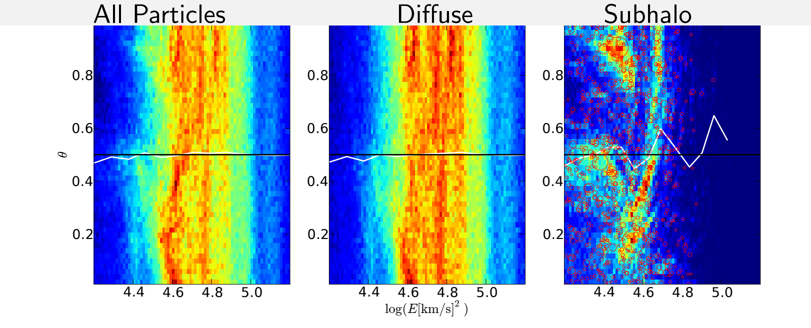

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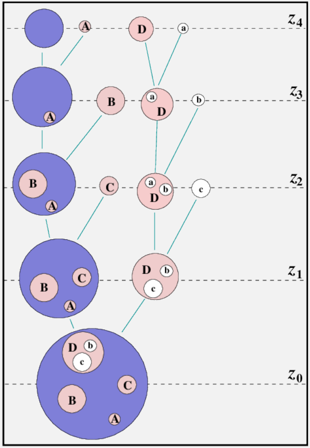
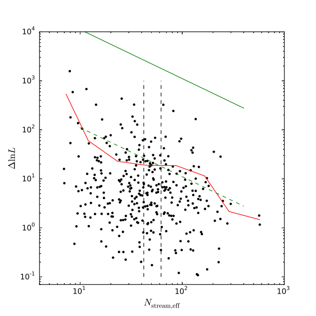


$$N_{\rm stream,eff} = \frac{(\sum n_i)^2}{\sum n_i^2} \in [1, m]$$



$$\Delta \ln L \sim \frac{N}{N_{\rm eff}} \chi^2(2)$$

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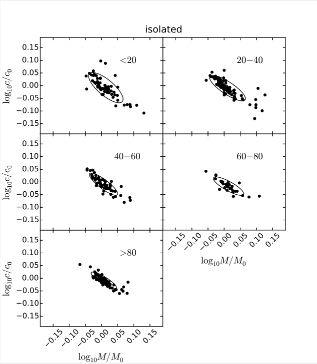

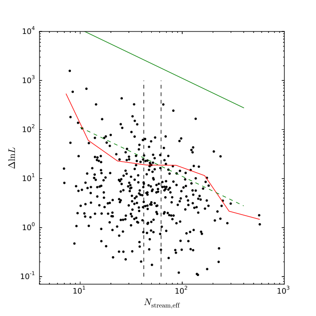

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$$\Delta \ln L \sim \frac{N}{N_{\mathrm{eff}}} \chi^2(2)$$

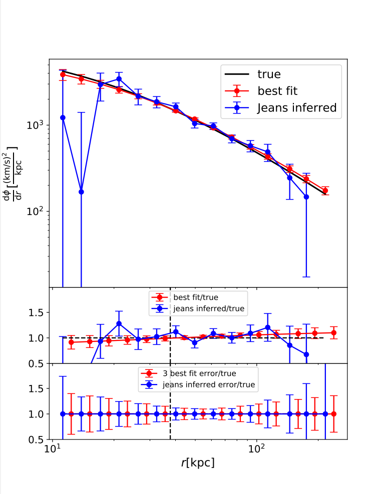
Independent test: The spherical Jeans equation

Taking the velocity moment ($\int \mathrm{d}^3 v \mathbf{v}[]$) of the collisionless Boltzmann equation

$$\mathbf{v} \cdot \frac{\partial f}{\partial x} - \frac{\partial \Phi}{\partial x}\frac{\partial f}{\partial v} = 0$$

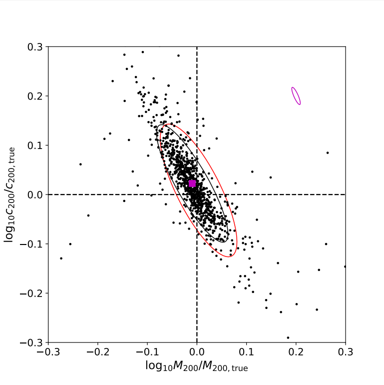
leads to the (spherical) Jeans equation:

$$\frac{\mathrm{d}(\rho \sigma_r^2)}{\mathrm{d}r} + 2\frac{\beta}{r}\rho\sigma_r^2 = -\rho\frac{\mathrm{d}\Phi}{\mathrm{d}r}$$

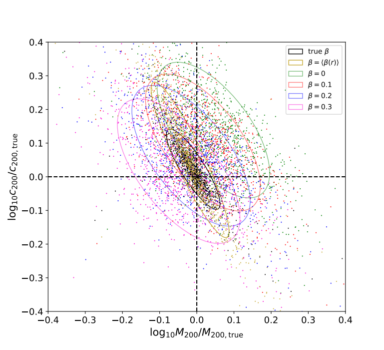


Alternative test using Jeans equation

Same systematic biases due to deviation from steady-state



Additional bias if $\beta(r)$ is unknown and assumed



(Wang,Han+18)

The amount of steady-state information is limited.